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VARIATIONAL MODEL OF ORGANIZED VORTICITY IN PLANE FLOW

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In the research of the last decade [mostly experimental (see the review [1]) and numerical (see the bibliography in [2])] a new phenomenon in turbulent flow has been widely studied: that of organized or coherent structures. The characteristic traits of coherent structures that are common in different flows have been formulated. In particular, the primary effect of nonviscous mechanisms on their formation and evolution have been noted. Hence, the analytical models of coherent structures use exact and approximate solutions of the Euler equations for the dynamics of an ideal fluid. However, this approach naturally forces various simplifications, and cannot completely take into account the existing information on coherent structures. For example, in models of shear layers [3-5], chains of coherent structures were considered with a uniform distribution of vorticity inside each of the individual structures. In [3, 5] coherent structures were represented by Kirchhoff and Rankine vortices. In [6-8] the equations for the chains of coherent structures were closed using circular vortices from a single-parameter family [9].

In most of the models of shear layers, the interaction of an individual structure with other coherent structures is taken into account approximately. For example, in [3] the effect of the chain was replaced by a uniform deformation field. In [4, 5] vortices of a given shape were used, and in [6-8] the simplest approximation of point vortices was used.

In the present paper an analytical model of coherent structures in plane flow is constructed by using a variational principle borrowed from information theory. The field of vorticity in the coherent structure is found from the condition that the informational entropy functional be a maximum. In this approach one can use additional constraints to take into account different kinds of information on the basic properties of coherent structures in specific examples, such as dynamical invariants, symmetry properties of the structures, and characteristics of the average flow field.

The variational principle is applied to the problem of a linear chain of coherent structures in an infinite shear layer. The functional equation for the vorticity field in an individual coherent structure is given, in which the nonviscous interactions of the structures are systematically taken into account. It is found that one of the analytical solutions of the equation can be represented in closed form. This is the single-parameter family of Stuart vortices [10]. Using this solution, we construct a model of a chain of coherent structures for a time-dependent shear layer and our model reproduces the general features of its evolution. It is shown that for a certain choice of the family parameter one can obtain, with the help of the Stuart vortices, certain average characteristics of turbulent mixing layers which correspond to experimental data satisfactorily.

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1. Numerous examples of the successful application of the principle of maximum informational entropy are known in various fields of physics (see the bibliography in [11, 12]). The principle allows one to construct objective estimates of physical fields which are consistent with the available information. Applied to the problem of the distribution of scalar vorticity $\Omega(R)$ (the z component of the vortex) in plane coherent structures and systems of plane coherent structures, the general method of applying the variational principle is explained in the following paragraphs.

With no loss of generality we can assume that the unknown vorticity $\Omega(R)$ is nonnegative and its total circulation is unity. We introduce the informational entropy functional

$$\widetilde{S}(\Omega) = -\int d\mathbf{R}\Omega(\mathbf{R}) \ln \Omega(\mathbf{R}). \tag{1.1}$$

Additional information on the properties of coherent structures is represented as a set of functionals of $\Omega(R)$ which are in general nonlinear

$$\varphi_i(\Omega) = \mu_i, \; i = 1, \ldots, N, \tag{1.2}$$

where the μ_i are assumed to be given. In the set of functionals (1.2) it is possible to include different dynamical invariants of the field $\Omega(R)$, its arbitrary moment characteristics, symmetry properties, features of the average flow, and other data.

The problem of finding the vorticity $\Omega(R)$ becomes a variational problem for the extremum of the functional

$$\widetilde{\Phi}(\Omega) = \widetilde{S}(\Omega) + \sum_{i=1}^{N} \lambda_i \varphi_i(\Omega), \tag{1.3}$$

where the λ_i are underdetermined Lagrange multipliers. From equating the variation of (1.3) to zero, certain functional equations follow, which together with the conditions (1.2) determine the vorticity.

In a certain sense this approach is similar to the method of [9, 13, 14], where equilibrium statistical mechanics was applied to dynamical systems of point (small [14]) vortices in an ideal fluid and certain equilibrium vorticity distributions were studied. Following a suggestion made in [6], these distributions began to be considered as possible vorticity fields in isolated coherent structures.

A complete comparison of the two approaches undoubtedly deserves special study. There are some advantages of the variational principle considered here. For example, the results of [9, 13, 14] (see also [15], where the vorticity potential was considered), deduced by the methods of statistical mechanics, can be obtained more simply and naturally from the variational principle with a minimum amount of calculation. Many of the steps necessary in constructing the statistical mechanics of the vorticity drop out of the problem: the transition to a dynamical system of point (discrete) vortices, the study of the first integrals of this system, the use of equilibrium chains for multivortex distribution functions [9] or the combinatorial derivation of the canonical distributions [13, 14], the transition to the single-vortex probability distribution density and its identification with the average continuous vorticity field, etc. As noted above, the conditions (1.2) can be of any nature and can, for example, include experimental information; thus it is possible to successively refine the model. In the statistical mechanics approach the additional conditions can only be additive integrals of the corresponding dynamical system. Use of the principle of maximum entropy allows one to discard the probabilistic treatment of the field, which is essential in the statistical mechanics approach. Finally, a restriction to the framework of the hydrodynamics of an ideal fluid ceases to be necessary.

2. The problem of the distribution of vorticity in a chain of coherent structures modeling an infinite shear layer if their nonviscous interactions are considered without additional simplifications [3-8] is an interesting problem of itself and can be used to demonstrate the possibilities of our method.

The chain of structures can represent a plane vortex flow, periodic in the direction of flow with period d. Let γ be the average circulation over a period. As a minimum system of functionals (1.2) for this problem, it is convenient to choose the circulation $\Gamma(\Omega) = \gamma d$ and the energy $\tilde{E}(\Omega)$ of an individual coherent structure. For the transition to dimensionless variables we use the quantities $d/2\pi$, $d/(4\pi^2\gamma)$, and $(\gamma d)^2$ for characteristic scales of length, vorticity, and energy, respectively. The density of the fluid is assumed to be unity.

The functional (1.3) is written in terms of dimensionless variables, as follows:

$$\Phi(\omega(\mathbf{r})) = S(\omega(\mathbf{r})) + \sigma - 4\pi\lambda E(\omega(\mathbf{r})).$$

Here σ and λ are undetermined Lagrange multipliers, and $\omega(\mathbf{r})$ is the vorticity in an individual structure with the normalization condition

$$\int_G d\mathbf{r}\omega(\mathbf{r}) = 1. \tag{2.1}$$

In general, the region of integration G is a certain curved strip with equidistant boundaries

$$G = \{\mathbf{r} = (x, y),\ \varphi(y) \leqslant x \leqslant \varphi(y) + 2\pi,\ \varphi(0) = -\pi,\ -\infty < y < \infty\}, \tag{2.2}$$

occupied by a single structure;

$$S(\omega(\mathbf{r})) = -\int_G d\mathbf{r}\omega(\mathbf{r}) \ln \omega(\mathbf{r}) \tag{2.3}$$

is the informational entropy functional of the vorticity $\omega(\mathbf{r})$:

$$E(\omega(\mathbf{r})) = \frac{1}{2}\int_G d\mathbf{r}\omega(\mathbf{r})\psi(\mathbf{r}) \tag{2.4}$$

is the "excess" energy of an individual coherent structure (the kinetic energy of the fluid in the strip G minus a divergent contour integral [16]). For brevity we simply refer to the quantity (2.4) as the energy.

The stream function $\psi(\mathbf{r})$ of the flow can be obtained by integrating the well-known expression [17]

$$\Psi(x, y) = -(4\pi)^{-1} \ln \frac{1}{2}[\operatorname{ch} y - \cos x] \tag{2.5}$$

for the stream function of a set of point vortices of identical circulation placed along the x axis with period 2π. The expression for $\psi(\mathbf{r})$ has the form

$$\psi(\mathbf{r}) = -(4\pi)^{-1}\int_G d\mathbf{r}_1\omega(\mathbf{r}_1) \ln \frac{1}{2}[\operatorname{ch}(y_1 - y_2) - \cos(x_1 - x_2)]. \tag{2.6}$$

Taking the first variation of $\Phi(\omega(\mathbf{r}))$ and setting the result equal to zero gives

$$\int_G d\mathbf{r}\delta\omega\{\sigma - 1 - \ln\omega(\mathbf{r}) - 4\pi\lambda\psi(\mathbf{r})\} = 0.$$

In view of the fact that $\delta\omega$ is arbitrary, it follows that

$$\omega(\mathbf{r}) = c\exp(-4\pi\lambda\psi(\mathbf{r})), \tag{2.7}$$

in which c is related in an obvious way with the Lagrange multiplier σ. Substitution of (2.6) into the right-hand side of (2.7) leads to an equation for the vorticity of the coherent structure:

$$\omega(\mathbf{r}) = c\exp\left\{\lambda\int_G d\mathbf{r}_1\omega(\mathbf{r}_1)\ln\frac{1}{2}[\operatorname{ch}(y - y_1) - \cos(x - x_1)]\right\}. \tag{2.8}$$

Equation (2.8), together with the normalization condition (2.1) and relation (2.4) (for a fixed value of E) form a system of equations for the vorticity $\omega(r)$ and the parameters c and λ.

3. The analysis establishes the following properties of the vorticity distribution satisfying (2.8), (2.4), and (2.1). It follows directly from (2.5) that inside the strip (2.2) $\Psi(x, y)$ is a harmonic function everywhere except at the point x = y = 0, where there is a point vortex of unit intensity. Hence,

$$\Delta\Psi(\mathbf{r}) = -\delta(\mathbf{r}).$$

Hence applying the Laplacian operator to (2.6), and using (2.7), it can be shown that the stream function for the vorticity from (2.8) satisfies the Liouville equation [10]

$$\Delta\psi = -c \exp(-4\pi\lambda\psi) \tag{3.1}$$

on the class of 2π-periodic functions in x. It is known [16] that for steady plane flow the stream function satisfies the nonlinear Poisson equation and (3.1) is a special case of this equation. This means that the vorticity distribution obtained from the variational principle is consistent with Euler's equations of inviscid hydrodynamics and determines a certain steady flow of an effectively nonviscous fluid.

In particular, for the case of a straight strip $F = \{r = (x, y), -\pi \le x \le \pi, -\infty < y < \infty\}$ the family of periodic solutions of (3.1) introduced by Stuart [10] are well known. The stream function and vorticity in this case have the form

$$\psi(x, y, \alpha) = -(4\pi)^{-1}\ln[\operatorname{ch} y - \alpha\cos x]; \tag{3.2}$$

$$\omega(x, y, \alpha) = -(4\pi)^{-1}(1-\alpha^2)[\operatorname{ch} y - \alpha\cos x]^{-2}, \tag{3.3}$$

where $0 \le \alpha \le 1$ is a parameter and $\alpha = 0$ corresponds to a shear layer with the velocity profile $u(y) = -(4\pi)^{-1}\operatorname{th} y$, while the limit $\alpha \to 1$ represents a regular chain of point vortices distributed along the x axis with period 2π. It can be verified directly that the vorticity (3.3) satisfies the normalization condition (2.1). Substitution of (3.2) and (3.3) into (2.7) shows that the Stuart solution can be obtained from the variational principle as the exact solution of (2.8) with

$$c(\alpha) = (4\pi)^{-1}(1-\alpha^2),\ \lambda = -2. \tag{3.4}$$

The asymptotic form of the stream function (2.6) is given by

$$\psi(x, y) \underset{y\to\pm\infty}{\simeq} -(4\pi)^{-1}\int_G d\mathbf{r}\omega(\mathbf{r})\ln\exp[\pm(y-y_1)] = -(4\pi)^{-1}|y|.$$

Hence it follows from (2.7) that in order for the total circulation of an individual coherent structure to be bounded it is necessary to require that $\lambda < 0$.

It is useful to introduce some expressions, which are usually calculated either with an information-theoretic approach [11], or with the use of statistical mechanics [9]. Substituting (2.7) into (2.3), and taking into account (2.1) and (2.4), we find an expression for the maximum of the entropy functional

$$S_m = -\ln c + 8\pi\lambda E. \tag{3.5}$$

As in [9] we can obtain an explicit expression for the Lagrange multiplier λ in this case. We differentiate the condition (2.1) with respect to E and substitute the expression for the vorticity from (2.7). Then we find

$$\frac{1}{c}\frac{\partial c}{\partial E} - 8\pi E\frac{\partial\lambda}{\partial E} - 4\pi\lambda\int_G d\mathbf{r}\omega(\mathbf{r})\frac{\partial\psi}{\partial E} = 0, \tag{3.6}$$

where we have used the normalization (2.1) and the definition (2.4) for E. It is evident from (3.6) that in the derivation of such differential relations we must remember that all quantities determined by the vorticity distribution are interrelated. It follows from (2.4) and (2.6) that

$$\int_G d\mathbf{r}\omega(\mathbf{r})\frac{\partial\psi}{\partial E} = 1.$$

Using this and (3.5) and (3.6), we obtain the required expression

$$\lambda = (4\pi)^{-1}\frac{\partial S}{\partial E}. \tag{3.7}$$

In analogy with a well-known thermodynamic relation (see also [9]), we note that the quantity λ^{-1} determines the "temperature" (the modulus of the distribution) of the vorticity of an individual coherent structure considered as a subsystem. Because for our model $\lambda < 0$, then, in contrast to [9], it follows from (3.7) that here the informational entropy functional falls off monotonically with increasing energy of the coherent structure. In other words, the informational measure of the coherence of the internal structure increases.

Several useful relations can be obtained if the quantities in the functional depend on a set of parameters. In particular, for the α-family of Stuart we have from (3.5) and (3.7) that

$$4\pi\lambda\frac{\partial E(\alpha)}{\partial\alpha} = \frac{\partial \ln c(\alpha)}{\partial\alpha}.$$

Substitution of the values $c(\alpha)$ and λ from (3.4) and integration leads to the expression

$$E(\alpha) = -(8\pi)^{-1}\ln(1-\alpha^2) + E(0).$$

The quantity $E(0)$ can be evaluated relatively simply [18]:

$$E(0) = -\frac{1}{2}(4\pi)^{-3}\int_G\int_G d\mathbf{r}_1 d\mathbf{r}_2 \operatorname{ch}^{-2}y_1 \operatorname{ch}^{-2}y_2 \times$$
$$\times \ln\left\{\frac{1}{2}[\operatorname{ch}(y_1-y_2) - \cos(x_1-x_2)]\right\} = -(8\pi)^{-1}(1-2\ln 2).$$

Then

$$E(\alpha) = -(8\pi)^{-1}\{\ln[(1-\alpha^2)/4] + 1\}. \tag{3.8}$$

We find the following expression for the informational entropy of the α-family from (3.4), (3.5), and (3.8)

$$S_m(\alpha) = \ln[(1-\alpha^2)\pi/4] + 2.$$

4. In analogy with [5-8], using the family of Stuart vortices, one can discuss a model of a chain of coherent structures which reproduces the pairing of vortices. We assume that after each act of pairing the steady-state distribution of vorticity in the coherent structure is described by (3.3) for a certain value of α. The distance d between the centers of neighboring vortices is then doubled. From the nonviscous conservation laws of vorticity and energy it follows that the total circulation of each coherent structure $\tilde{\Gamma} = \gamma d$ is also doubled, and its dimensionless energy $E(\alpha)$ increases. As seen from (3.8), $E(\alpha)$ is a monotonically increasing function of the parameter α which therefore also increases in the process of pairing.

This model qualitatively reflects several observable physical effects [1]: the evolution by pairing of vortices, the primarily nonviscous nature of their interaction, the transport of energy into large-scale motion, the universality of the distribution of vorticity in coherent structures, and a definite symmetry, in which their mutual effect is taken into account. Because $\lambda = -2$, it is evident from (3.7) that in the process of pairing the informational entropy (2.3) decreases with an increase in the energy of the structures, i.e., the model contains the effect of increasing order (coherence) inside the individual vortices, which is also observed in experiment. It would seem that this property contradicts the principle of maximum informational entropy applied to the chain of coherent structures as a whole. The resolution of this apparent paradox is the dimensional nature of the effect (compare [19]). For the assumed system the entropy functional (1.1) in dimensional form can be written as

$$\tilde{S}(\alpha) = S_m(\alpha) + 2\ln(d/2\pi).$$

Using (3.8) and (3.9), this relation can be rewritten in the form

$$\tilde{S}(\alpha) = -8\pi E(\alpha) + \ln[ed^2/4\pi]. \tag{4.1}$$

The conservation of energy for a given state of the chain is expressed by the relation

$$E(\alpha) = E_0. \tag{4.2}$$

Here E_0 is the energy of the fluid in the region G in the unperturbed flow, which is the initial state for the cascade of pairings. Usually it can be assumed that in the initial shear flow the distribution of vorticity depends only on the transverse coordinate: $\omega(\mathbf{r}) = (4\pi\delta)^{-1}\omega(y/\delta)$, where δ is the characteristic thickness of the layer. Then the normalization condition (2.1) transforms into the relation

$$(2\delta)^{-1}\int_{-\infty}^{\infty} dy\omega(y/\delta) = 1,$$

and then one can write

$$E_0 = (4\pi)^{-1}\ln 2 + \delta E_\delta, \tag{4.3}$$
$$E_\delta = -(8\pi)^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz_1 dz_2 \omega(z_1)\omega(z_2)|z_1 - z_2|,$$

where we have introduced the new variable $z = y/\delta$ and we have used the relation

$$\int_0^{2\pi} dx \ln[a^2 + b^2 - 2ab\cos x] = 2\pi\ln(\max(a^2, b^2)).$$

Using (4.2) and (4.3), it follows from (4.1) that

$$\tilde{S}(d) = -16\pi^2\frac{\tilde{\delta}}{d}E_\delta + 2\ln\left(\frac{d}{4}\sqrt{\frac{e}{\pi}}\right). \tag{4.4}$$

It is evident from this relation that the dimensional entropy functional depends only on the distance d between the centers of neighboring structures, and this relation is satisfied after each pairing. The function $\tilde{S}(\alpha)$ has a unique minimum at the point $d_0 = -16\pi^2\tilde{\delta}E_\delta$ and monotonically increases for $d > d_0$. In terms of dimensionless variables, the equivalent inequality is $8\pi\delta E_\delta > -1$. From the law of conservation of energy (4.2), it can be shown that this relation is necessarily satisfied if after the first pairing $0 < \alpha < 1$. This means that the entropy functional (4.1) increases as the model chain evolves.

In order to estimate the coefficient of alternation for our model we use its definition in the form

$$\chi = 2\pi/\bar{\delta}_\omega, \quad \bar{\delta}_\omega^{-1} = \frac{1}{2\pi}\int_{-\pi}^{\pi} dx\omega(x, 0). \tag{4.5}$$

Using the normalization (2.1), it can be seen that (4.5) is constructed on a shear layer of effective thickness $\delta_\omega = \Delta U/|\partial u/\partial y|_{max}$, averaged with respect to the longitudinal coordinate over the extent of a single structure. Substitution of (3.3) into (4.5) gives an expression for the coefficient of alternation in terms of the parameter α

$$\chi(\alpha) = \pi/(1 - \alpha^2)^{1/2}, \tag{4.6}$$

and the relation (4.2) can be used to express it in terms of the energy of the initial state

$$\chi = \pi\exp\left[\frac{1}{2}(1 + 8\pi\delta E_\delta)\right].$$

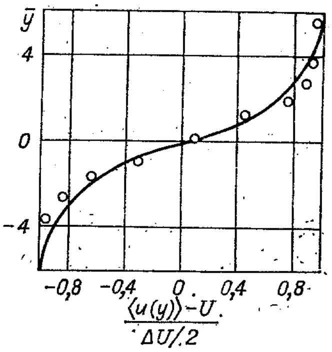

Fig. 1

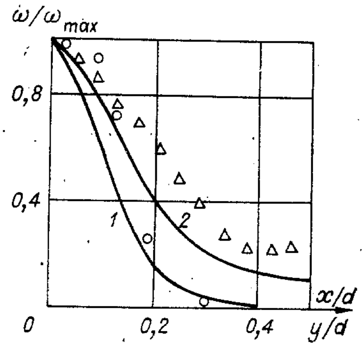

Fig. 2

In [6-8] the value of χ was calculated for the self-modeling state of the chain, in which the original thickness of the shear layer is negligibly small in comparison with the distance between coherent structures; this corresponds to the limit $\delta \to 0$. In this case, the last equation and (4.6) give the self-modeling value for the chain of Stuart vortices

$$\chi_a = \pi\sqrt{e} \simeq 5{,}2,\ \alpha_a = (1 - e^{-1})^{1/2} \simeq 0{,}8.$$

The value χ_a lies on the upper boundary of the range of experimental values [1].

5. It is interesting to consider if the family of Stuart vortices can be used to model other turbulent characteristics of shear layers with coherent structures. If we neglect small-scale fluctuations, then using (3.2) the local instantaneous velocities along and transverse to the flow can be expressed by the equations

$$u(x, y, t) = U + \frac{1}{2}\,\mathrm{sh}\,y\,[\mathrm{ch}\,y - \alpha\cos Ut]^{-1}\,\Delta U/2,$$
$$v(x, y, t) = \sin Ut[\mathrm{ch}\,y - \alpha\cos Ut]^{-1}\Delta U/2, \qquad (5.1)$$

where $U = (U_+ + U_-)/2$; $\Delta U = U_+ - U_-$; $U_\pm$ are the asymptotic values of the velocity at the external boundaries of the shear layer (mixing layer). Starting from (5.1) and with the help of an average over the time interval $0 \le t \le \pi/U$, we can obtain expressions for various characteristics of the turbulent flow containing the single free parameter α. Its value can be chosen from the best approximation of the mean velocity profile and one can analyze how the model then reproduces the other turbulent quantities. For the above mean velocity profile we have from (5.1)

$$\frac{\langle u\rangle - U}{\Delta U/2} = \frac{\mathrm{sh}\,y}{[\mathrm{ch}^2\,y - \alpha]^{1/2}}. \qquad (5.2)$$

In Fig. 1 we show a comparison of the profile (5.2) (line) with the experimental data of [20] (points). For the choice α = 0.49 the mean relative deviation lies within 3% and the maximum deviation does not exceed 10%, i.e., the deviation is the same as for more complicated models of [20].

In Fig. 2 the curves 1 and 2 give the vorticity distribution obtained from (3.3):

$$\omega(x, y, \alpha)/\omega(0, 0, \alpha) = (1 - \alpha)^2/(\mathrm{ch}\,y - \alpha\cos x)^2,\ \alpha = 0{,}49$$

for x = 0 and y = 0, and the corresponding experimental values of [20] are denoted by the circles and triangles.

Although the deviation here is 50%, one can claim a satisfactory qualitative agreement.

The value of the coefficient of alternation (4.6) is equal to χ = 3.6 for α = 0.49, and this is sufficiently close to the value χ ≃ 3.2 measured in [20].

The averaging procedure gives the following results for the mean-square transverse and longitudinal fluctuations

$$\langle v'^2\rangle^{1/2}/\Delta U = \frac{1}{2\alpha}[\mathrm{ch}y/R - 1]^{1/2},$$

$$\langle u'^2\rangle/\Delta U = (^1/_2)|\mathrm{sh}\,y|/R\,[\mathrm{ch}\,y/R + \mathrm{sh}^2 y - (1 + \alpha^2)]^{1/2},$$

$$R = (\mathrm{ch}^2 y - \alpha^2)^{1/2},\ \alpha = 0{,}49.$$

For $\alpha = 0.49$, the transverse fluctuation profile qualitatively corresponds to the experimental results of [20], although the discrepancy reaches 100%. For the longitudinal fluctuations an asymmetric profile with a maximum near the x axis is experimentally established. But from the equations given above it is evident that the model profile is symmetric with respect to the longitudinal axis and the fluctuations go to zero at $y = 0$. This contradiction is also obtained for the Reynolds stress profile and is due to the symmetry of the Stuart vortices with respect to both axes.

We note that overall the model of coherent structures introduced here is more complete and systematic than previous models [3, 6-8]. There is the expectation that the numerical study of (2.8) will allow one to construct models of chains of coherent structures with other types of symmetry which will satisfactorily reproduce the fluctuation characteristics in shear layers.

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